This unit is an extension of previous work on both Geometry and Coordinate Geometry. No new content, but higher level applications of known concepts.
Syllabus references 6.8, 2.5
Coordinate Methods in Geometry and Applications of Coordinate Geometry

How would you show:
• A group of three points on the Cartesian plane makes an isosceles triangle?
• A figure on the Cartesian plane is a parallelogram?
• How to find the fourth point in a parallelogram, given the other three points?
• That four points lie on a circle?
• That a triangle is right angled?
• That two lines meet at right angles?
• That two shapes are similar?
• That two shapes are exactly the same?
Coordinate Methods in Geometry and Applications of Coordinate Geometry

Useful Formulae:
distance between two points:
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]

gradient of a line segment joining two points
\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
when lines are parallel, \( m_1 = m_2 \)
when lines are perpendicular \( m_1 m_2 = -1 \)

perpendicular (shortest) distance from a point to a line
\[ d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \]
equation of a line if you know the gradient and one point
\[ y - y_1 = m(x - x_1) \]

Useful terms:
A **median** of a triangle joins a vertex to the midpoint of the opposite side
An **altitude** of a triangle is the perpendicular from a vertex to the opposite side.
Warm up...Cambridge Ex 8A

9. (a) Show that $OC$ is not perpendicular to $OA$.
   
   (b) Show that $OD$ is not perpendicular to $OA$.
   
   (c) Show that $A, O$ and $C$ are not collinear.
   
   (d) Show that $A, O$ and $D$ are not collinear.

10. Find $\theta$ and $\phi$ in the diagrams below, giving reasons.
   
   (a) 
   
   (b) 
   
   (c) 
   
   (d) 

11. (a) Name all straight angles and vertically opposite angles in the diagram.
   
   (b) Which two lines in the diagram above are parallel?
   
   (c) Which two lines in the diagram above form a right angle?

12. Find the angle $\alpha$ in each diagram below.
   
   (a) 
   
   (b) 
   
   (c)
13. (a) \[ \gamma = 180^\circ - (\alpha + \beta). \]

(b) \[ \gamma = \alpha + \beta. \]

(c) \[ \gamma = \alpha - \beta. \]

(d) \[ EF \parallel AB. \]

14. **Theorem:** The bisectors of adjacent supplementary angles form a right angle. In the diagram to the right, \( \angle ABD \) and \( \angle DBC \) are adjacent supplementary angles. Given that the line \( FB \) bisects \( \angle DBC \) and the line \( EB \) bisects \( \angle ABD \), prove that \( \angle FBE = 90^\circ \).

15. In the diagram to the right, the line \( CO \) is perpendicular to the line \( AO \), and the line \( DO \) is perpendicular to the line \( BO \). Show that the angles \( \angle AOD \) and \( \angle BOC \) are supplementary.
Cambridge Ex 8B

8. Find the angles $\alpha$ and $\beta$ in the diagrams below. Give all steps in your argument.

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

9. Find the angles $\theta$ and $\phi$ in the diagrams below, giving all reasons.

(a) 

(b) 

(c) 

(d)
10. Find the value of $\alpha$ in the diagrams below, giving all reasons.

(a) \[\triangle ABC\] with $\angle A = 45^\circ$, $\angle B = 66^\circ$, $\angle C = 2\alpha^\circ$.

(b) \[\triangle ABC\] with $\angle A = 3\alpha + 12^\circ$, $\angle B = 5\alpha - 11^\circ$, $\angle C = 43^\circ$.

(c) \[\triangle ABC\] with $\angle A = 3\alpha - 15^\circ$, $\angle B = 2\alpha + 23^\circ$, $\angle C = 118^\circ$.

(d) \[\triangle ABC\] with $\angle A = 2\alpha + 9^\circ$, $\angle B = 2\alpha - 12^\circ$, $\angle C = 41^\circ$.

(e) \[\triangle ABC\] with $\angle A = \alpha + 13^\circ$, $\angle B = \alpha + 1^\circ$, $\angle C = 4\alpha - 16^\circ$.

(f) \[\triangle ABC\] with $\angle A = 4\alpha - 29^\circ$, $\angle B = 104^\circ$, $\angle C = 3\alpha - 30^\circ$.

(g) \[\triangle ABC\] with $\angle A = \alpha + 40^\circ$, $\angle B = \alpha + 50^\circ$, $\angle C = \alpha + 60^\circ$.

(h) \[\triangle ABC\] with $\angle A = \alpha + 30^\circ$, $\angle B = \alpha + 60^\circ$, $\angle C = \alpha + 50^\circ$.

11. Find the values of $\alpha$ in the diagrams below, giving all reasons.

(a) \[\pentagon ABCDE\] with $\angle A = 110^\circ$, $\angle B = 120^\circ$.

(b) \[\pentagon ABCDE\] with $\angle A = 2\alpha$, $\angle B = 104^\circ$, $\angle C = 125^\circ$, $\angle D = 107^\circ$.

(c) \[\pentagon ABCDE\] with $\angle A = 2\alpha - 16^\circ$, $\angle B = 77^\circ$, $\angle C = 3\alpha$, $\angle D = 4\alpha - 53^\circ$.

12. Prove the given relationships in the diagrams below.

(a) \[\triangle ABC\] with $\angle A + \beta = \alpha$, $\alpha + \beta = 90^\circ$.

(b) \[\triangle ABC\] with $\angle A = \beta$, $\alpha + \beta = 90^\circ$.

(c) \[\triangle ABC\] with $\angle A = 2\alpha$, $\angle B = 3\beta$, $\beta = 72^\circ$, $\alpha = 36^\circ$.

(d) \[\square ABCD\] with $\angle A = \alpha$, $\angle B = \beta$, $\angle D = \alpha$, $\angle C = \beta$, $AB \parallel CD$ and $AD \parallel BC$. 
13. Course Theorem: An alternative proof of the exterior angle theorem. Given a triangle $ABC$ with $BC$ produced to $D$, construct the line $XY$ through the vertex $A$ parallel to $BD$. Let $\angle CAB = \alpha$ and $\angle ABC = \beta$. Use alternate angles twice to prove that $\angle ACD = \alpha + \beta$.

14. Course Theorem: An alternative proof that the angle sum of a triangle is $180^\circ$. Let $ABC$ be a triangle with $BC$ produced to $D$. Construct the line $CE$ through $C$ parallel to $BA$. Let $\angle CAB = \alpha$, $\angle ABC = \beta$ and $\angle BCA = \gamma$. Prove that $\alpha + \beta + \gamma = 180^\circ$.

15. Course Theorem: An alternative approach to proving that the angle sum of a quadrilateral is $360^\circ$.

(a) Suppose that a quadrilateral has a pair of parallel sides, and name them $AB$ and $CD$ as shown. Use the assumptions about parallel lines and transversals to prove that the interior angle sum of quadrilateral $ABCD$ is $360^\circ$.

(b) Suppose that in quadrilateral $ABCD$ there is no pair of parallel sides. Extend sides $AB$ and $DC$ to meet at $E$ as shown. Use the theorems about angles in triangles to prove that the interior angle sum of quadrilateral $ABCD$ is $360^\circ$. 
Cambridge Ex 8C

5. (a) When asked to show that the two triangles above were congruent, a student wrote \( \triangle RST \equiv \triangle UVW \) (RHS). Although both triangles are indeed right-angled, explain why the reason given is incorrect. What is the correct reason?

(b) When asked to show that the two triangles above were congruent, another student wrote \( \triangle GHI \equiv \triangle ABC \) (RHS). Again, although both triangles are right-angled, explain why the reason given is wrong. What is the correct reason?

6. In each part, prove that the two triangles in the diagram are congruent.

(a) \( \triangle ABC \) \( \triangle CDA \)

(b) \( \triangle BCD \) \( \triangle ACD \)

(c) \( \triangle CBD \) \( \triangle ABD \)

(d) \( \triangle Def \) \( \triangle ABC \)

7. Let \( M \) be any point on the base \( BC \) of an isosceles triangle \( ABC \). Using the facts that the legs \( AB \) and \( AC \) are equal, the base angles \( \angle B \) and \( \angle C \) are equal, and the side \( AM \) is common, is it possible to prove that the triangles \( ABM \) and \( ACM \) are congruent?

11. (a) Given that \( \triangle ABD \equiv \triangle CDB \) in the diagram above, prove that \( \triangle BDE \) is isosceles.

(b) If \( DM = MB \) and \( AC \perp DB \), prove that \( \triangle ABD \) and \( \triangle CBD \) are isosceles.
16. In the diagram, \( AB \parallel DC \) and \( \angle CAB = \angle ABD = \alpha \).
   (a) Show that \( CE = DE \).
   (b) Prove that \( \triangle ABC \cong \triangle BAD \).
   (c) Hence show that \( \angle DAC = \angle CBD \).

17. Triangle \( ABC \) has a right angle at \( B \), \( D \) is the
   midpoint of \( AB \), and \( DE \) is parallel to \( BC \).
   (a) Prove that \( \angle ADE \) is a right angle.
   (b) Prove that \( \triangle AED \cong \triangle BED \).
   (c) Prove that \( BE = EC \).

18. The diagonals \( AC \) and \( DB \) of quadrilateral \( ABCD \) are equal
   and intersect at \( X \). Also, \( AD = BC \).
   (a) Show that \( \triangle ABC \cong \triangle BAD \).
   (b) Hence show that \( \angle ABCX \) is isosceles.
   (c) Thus show that \( \triangle CDX \) is also isosceles.
   (d) Show that \( AB \parallel DC \).

19. (a)

   In the diagram, \( \triangle PQR \) is isosceles with \( PQ = PR \), and \( \angle PQR = 48^\circ \).
   The interval \( QR \) is produced to \( S \). The bisectors of
   \( \angle PQR \) and \( \angle PRS \) meet at the point \( T \).
   (i) Find \( \angle PQR \).
   (ii) Find \( \angle QTR \).

   (b)

   In \( \triangle ABC \), \( AB \) is produced to \( D \), \( AC \) bisects \( \angle CAB \) and \( BE \) bisects \( \angle CBD \).
   (i) If \( \angle ABE \) is isosceles with \( \angle A = \angle E \), show that \( \triangle ABC \) is also isosceles.
   (ii) If \( \angle ABC \) is isosceles with \( \angle A = \angle B \), under what circumstances will \( \triangle ABE \)
   be isosceles?

20. (a)

   The bisector of \( \angle RAC \) meets \( BC \) at \( Y \).
   The point \( X \) is constructed on \( AY \) so that
   \( \angle ABX = \angle ACB \). Prove that \( \triangle BXY \) is
   isosceles.

   (b)

   The diagonals \( AC \) and \( BD \) of quadrilateral
   \( ABCD \) meet at right angles at \( X \). Also,
   \( \angle ADX = \angle CDX \).
   (i) Prove that \( AD = CD \).
   (ii) Hence prove that \( AB = CB \).

21. In \( \triangle ABC \), \( \angle CAB = \angle CBA = \alpha \). Construct \( D \) on \( AB \)
   and \( E \) on \( CB \) so that \( CD = CE \). Let \( \angle ACD = \beta \).
   (a) Explain why \( \angle CDB = \alpha + \beta \).
   (b) Find \( \angle DCB \) in terms of \( \alpha \) and \( \beta \).
   (c) Hence find \( \angle EDB \) in terms of \( \beta \).

22. Theorem: The line of centres of two intersecting circles is
    the perpendicular bisector of the common chord.
    The diagram to the right shows the two circles intersecting at \( A \)
    and \( B \). The line of centres \( OP \) intersects \( AB \) at \( M \).
    (a) Explain why \( \triangle ABO \) and \( \triangle ABP \) are isosceles.
    (b) Show that \( \triangle AOP \cong \triangle BOP \).
    (c) Show that \( \triangle AOM \cong \triangle BMO \).
    (d) Hence show that \( AM = BM \) and \( AB \perp OP \).
23. **Pentagons and Trigonometry:** \( ABCDE \) is a regular pentagon with side length \( x \). Each interior angle is \( 108^\circ \).
   (a) State why \( \triangle ABC \) is isosceles and find \( \angle CAB \).
   (b) Show that \( \triangle ABC \cong \triangle DEA \).
   (c) Find \( \angle CAD \).
   (d) Find an expression for the area of the pentagon in terms of \( x \) and trigonometric ratios.

25. **The Circumcentre Theorem:** The perpendicular bisectors of the sides of a triangle are concurrent, and the resulting circumcentre is the centre of the circumcircle through all three vertices. Let \( P \), \( Q \) and \( R \) be the midpoints of the sides \( BC \), \( CA \) and \( AB \) of \( \triangle ABC \). Let the perpendiculars from \( Q \) and \( R \) meet at \( O \), and join \( OA \), \( OB \), \( OC \) and \( OP \).
   (a) Prove that \( \triangle ORA \cong \triangle ORB \).
   (b) Prove that \( \triangle OQA \cong \triangle OQC \).
   (c) Hence prove that \( OA = OB = OC \), and \( OP \perp BC \).

26. **A Geometric Inequality:** The angle opposite a longer side of a triangle is larger than the angle opposite a shorter side. Suppose that \( \triangle ABC \) is a triangle in which \( CA > CB \). Construct \( P \) between \( C \) and \( A \) so that \( CP = CB \), and let \( \alpha = \angle A \) and \( \theta = \angle CPB \).
   (a) Explain why \( \alpha < \theta \). (b) Explain why \( \angle CBP = \theta \).
   (c) Hence prove that \( \alpha < \angle CBA \).

27. **A Rotation Theorem:** The triangles \( OAB \) and \( OCD \) in the figure drawn to the right are both equilateral triangles, and they have a common vertex \( O \). Prove that \( AC = BD \).
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8. Properties of a parallelogram: In this question, you must use the definition of a parallelogram as a quadrilateral in which the opposite sides are parallel.

(a) **Course Theorem:** Adjacent angles of a parallelogram are supplementary, and opposite angles are equal.

The diagram shows a parallelogram $ABCD$. Explain why $\angle A + \angle B = 180^\circ$ and $\angle A = \angle C$.

(b) **Course Theorem:** Opposite sides of a parallelogram are equal. The diagram shows a parallelogram $ABCD$ with diagonal $AC$.

(i) Prove that $\triangle ACB \cong \triangle CAD$.

(ii) Hence show that $AB = DC$ and $BC = AD$.

(c) **Course Theorem:** The diagonals of a parallelogram bisect each other. The diagram shows a parallelogram $ABCD$ with diagonals meeting at $M$.

(i) Prove that $\triangle ABM \cong \triangle CDM$ (use part (b)).

(ii) Hence show that $AM = MC$.

9. Tests for a parallelogram: These four theorems give the standard tests for a quadrilateral to be a parallelogram.

(a) **Course Theorem:** If the opposite angles of a quadrilateral are equal, then it is a parallelogram.

The diagram opposite shows a quadrilateral $ABCD$ in which $\angle A = \angle C = \alpha$ and $\angle B = \angle D = \beta$.

(i) Prove that $\alpha + \beta = 180^\circ$.

(ii) Hence show that $AB \parallel DC$ and $AD \parallel BC$. 

11. In the diagram, $ABCD$ is a parallelogram. The points $X$ and $Y$ lie on $BC$ and $AD$ respectively such that $BX = DY$.
(a) Explain why $\triangle ABX \cong \triangle CDY$.
(b) Explain why $AB = CD$.
(c) Show that $\triangle ABX \cong \triangle CDY$.
(d) Hence prove that $AYCX$ is a parallelogram.

12. The diagram shows the parallelogram $ABCD$ with diagonal $AC$. The points $P$ and $Q$ lie on this diagonal in such a way that $AP = CQ$.
(a) Prove that $\triangle ABP \cong \triangle CDQ$.
(b) Prove that $\triangle ADP \cong \triangle CBQ$.
(c) Hence prove that $BQDP$ is a parallelogram.

13. The diagram shows the parallelogram $ABCD$ and points $X$ and $Y$ on $AB$ and $CD$ respectively, with $AX = CY$. The diagonal $AC$ intersects $XY$ at $Z$.
(a) Prove that $\triangle AXZ \cong \triangle CYZ$.
(b) Hence prove that $XY$ is concurrent with the diagonals.

17. The diagonals of quadrilateral $ABCD$ meet at $M$, and $\triangle ABM \cong \triangle DCM$.
(a) Draw a diagram showing this information.
(b) Prove that $ABCD$ is a trapezium with equal base angles.

20. **Theorem:** The quadrilateral formed by joining the midpoints of the sides of a quadrilateral is a parallelogram. In quadrilateral $ABCD$, the points $Q$, $R$ and $S$ are the midpoints of $BC$, $CD$ and $DA$ respectively. The two points $P$ and $T$ lie on $AB$ and $AC$ respectively such that $PT = BQ$ and $PT \parallel BQ$.
(a) Explain why $PBQT$ is a parallelogram.
(b) Show that the four triangles $\triangle APT$, $\triangle QPT$, $\triangle PBQ$ and $\triangle TQC$ are all congruent, and that $P$ is the midpoint of $AB$.
(c) Hence show that the line joining the midpoints of two adjacent sides of a quadrilateral is parallel to the diagonal joining those two sides.
(d) Hence show that $PQRS$ is a parallelogram.
10. (a) The point $E$ is the midpoint of the side $CD$ of the rectangle $ABCD$.
   (i) Prove that $\triangle BCE \cong \triangle ADE$.
   (ii) Hence show that $\triangle ABE$ is isosceles.

15. (a) $P$ and $Q$ lie on the diagonal $BD$ of square $ABCD$, and $BP = DQ$. 
   (i) Prove that $\triangle ABP \cong \triangle CQP \cong \triangle ADQ \cong \triangle CDQ$.
   (ii) Hence show that $APCQ$ is a rhombus.

16. The parallelogram $PQRS$ is inscribed in $\triangle PBA$ with $R$ on $AB$. It is found that $QA = QR$ and $PS = SB$.
   (a) Prove that $\triangle BSR \cong \triangle RQA$.
   (b) Hence prove that $PQRS$ is a rhombus.

17. In the square $ABCD$, $P$ is on $AB$, $Q$ is on $BC$ and $R$ is on $CD$, with $AP = BQ = CR$.
   (a) Prove that $\triangle PBQ \cong \triangle QCR$.
   (b) Prove that $\angle PQR$ is a right angle.
18. In the rhombus $ABCD$, $AP$ is constructed perpendicular to $BC$ and intersects the diagonal $BD$ at $Q$.

(a) State why $\angle ADB = \angle CDB$.

(b) Prove that $\triangle AQP \cong \triangle CQD$.

(c) Show that $\angle DAQ$ is a right angle.

(d) Hence find $\angle QCD$.

19. The triangles $ABC$ and $APR$ are both right-angled at the vertices marked in the diagram. The midpoint of $PR$ is $Q$, and it is found that $PQ = QR = AB$.

(a) Explain why $\angle PBC = \angle PRA$.

(b) Construct the point $S$ that completes the rectangle $APSR$. Explain why $Q$ is also the midpoint of $AS$ and why $PQ = AQ$.

(c) Hence prove that $\angle PBA = 2 \times \angle PBC$. 


8. Prove that the four small triangles formed by the two diagonals of a parallelogram all have the same area. Under what circumstances are they all congruent?

9. The diagonals of a parallelogram form the diameters of two circles.
   (a) Why are they concentric?
   (b) If the diagonals are in the ratio \( a : b \), what is the ratio of the areas of the circles?
   (c) Under what circumstances do the circles coincide?

10. In the diagram to the right, \( ABCD \) and \( PQRS \) are squares, and \( AB = 1 \) metre. Let \( AP = x \).
    (a) Find an expression for the area of \( PQRS \) in terms of \( x \).
    (b) What is the minimum area of \( PQRS \), and what value of \( x \) gives this minimum?
    (c) Explain why the result is the same if the total area of the four triangles is maximised.

11. The diagram shows a rectangle with a square offset in one corner. All dimensions shown are in metres.
    (a) Find the area of the square.
    (b) Hence find the shaded area outside the square.

12. (a) The diagram shows a regular hexagon inscribed in a circle of radius 1 and centre \( O \).
    (i) Find the area of \( \triangle AOB \).
    (ii) Hence find the area of the hexagon.
    (b) The second diagram on the right shows another regular hexagon inscribed around a circle of radius 1, that is, each side is tangent to the circle.
    (i) Find the area of \( \triangle OGH \).
    (ii) Find the area of the hexagon.
    (c) Hence explain why \( \frac{3\sqrt{3}}{2} < \pi < 2\sqrt{3} \).

13. In the diagram opposite, \( ABCD \) and \( BFDH \) are congruent rectangles with \( AB = 8 \) and \( BC = 6 \).
    (a) Explain why \( \triangle ADG \equiv \triangle HBG \).
    (b) Show that \( AG = \frac{AB^2 - AD^2}{2AB} \) by using Pythagoras’ theorem, and hence find \( AG \).
    (c) Hence find the area of \( BEDG \).
2. Find $\alpha$, $\beta$, $\gamma$ and $\delta$. In parts (g) and (h), prove that $\text{arc } ABC = \text{arc } BCD$ and $AC = BD$.

(a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d)

(e) \hspace{1cm} (f) \hspace{1cm} (g) \hspace{1cm} (h)

3. (a) \hspace{1cm} (b) \hspace{1cm} (c)

Find $AO$.

Find $FH$ and $\cos \alpha$.

Find $OQ$, $QR$ and $\cos \alpha$. 
6. **Course Theorem:** Equal chords subtend equal angles at the centre, and are equidistant from the centre.
   In the diagram opposite, $AB$ and $XY$ are equal chords.
   (a) Prove that $\triangle AOB \cong \triangle XOY$.
   (b) Prove that $\angle AOB = \angle XOY$.
   (c) Prove that the chords are equidistant from the centre.

7. **Course Theorem:** Two chords subtending equal angles at the centre have equal lengths.
   In the diagram opposite, the angles $\angle AOB$ and $\angle XOY$ subtended by $AB$ and $XY$ are equal.
   (a) Prove that $\triangle AOB \cong \triangle XOY$.
   (b) Hence prove that $AB = XY$.

8. **Course Theorem:** Two chords equidistant from the centre have equal lengths.
   In the diagram opposite, $OM = ON$.
   (a) Prove that $\triangle OAM \cong \triangle OXN$.
   (b) Prove that $\triangle OBM \cong \triangle OYN$.
   (c) Hence prove that $AB = XY$.

10. (a) Prove that $\angle POG = 3\beta$.
    (b) Prove that $\angle TOY = \theta$.
    (c) Prove that $OD \parallel AP$.

11. (a) Prove that $AF = BG$.
    [HINT: First prove that $\triangle OAF \cong \triangle OBG$.]
    (b) Prove that $AF = BF$.
    [HINT: First prove that $\triangle OAF \cong \triangle OBF$.]
    (c) Prove that $AF = BG$.
    [HINT: First use intercepts to prove that $FO = OG$.]
12. (a) Prove that $FJ = KG$, and that $MG = MJ$. (b) Prove that $\angle PAB = \angle QAB$, and that $AB$ is a diameter. (c) Prove that $SP = SQ$, and that $PQ \perp ST$.

13. **Theorem:** When two circles intersect, the line joining their centres is the perpendicular bisector of the common chord. In the diagram opposite, two circles intersect at $A$ and $B$.
   (a) Prove that $\triangle OAP \equiv \triangle OBP$.
   (b) Hence prove that $\triangle OMA \equiv \triangleOMB$.
   (c) Hence prove that $AM = MB$ and $AB \perp OP$.
   (d) Under what circumstances will $OAPB$ form a rhombus?

14. In the configuration of the previous question, suppose also that each circle passes through the centre of the other (the circles will then have the same radius).
   (a) Prove that the common chord subtends $120^\circ$ at each centre.
   (b) Find the ratio $AB : OP$.
   (c) Use the formula for the area of the segment to find the ratio of the overlapping area to the area of circle $C$.

15. **Theorem:** If an isosceles triangle is inscribed in a circle, then the line joining the apex and the centre is perpendicular to the base. In the diagram opposite, $CA = CB$.
   (a) Prove that $\angle CAO = \angle CBO$ and $\angle ACM = \angle BCM$.
   (b) Hence prove that $COM \perp AB$. 
As you already know, exactly one circle can be drawn through any three non-collinear points. We can find the equation of this circle by finding the perpendicular bisectors of any two intervals joining the points. Where they intersect is the centre of the circle (called the circumcentre of the circle). The radius can then be found using the distance formula.

**Example:** take the points A (3,0), B (8,1) and C (6,2)

\[ m_{AB} = \frac{1}{5} \text{ so perpendicular gradient is } -5 \]
\[ \text{midpoint}_{AB} = (5.5,0.5) \]
\[ \text{equation of perpendicular bisector} \]
\[ y-y_1 = m(x-x_1) \]
\[ y-0.5 = -5(x - 5.5) \]
\[ y = -5x + 28 \text{ i)} \]

\[ m_{AC} = \frac{2}{3} \text{ so perpendicular gradient is } -\frac{3}{2} \]
\[ \text{midpoint}_{AB} = (4.5,1) \]
\[ \text{equation of perpendicular bisector} \]
\[ y-y_1 = m(x-x_1) \]
\[ y - 1 = -\frac{3}{2}(x - 4.5) \]
\[ y = -\frac{3}{2}x +\frac{31}{4} \text{ ii)} \]

Solving i) and ii) simultaneously gives the point \((-3 \frac{3}{26}, 12 \frac{11}{26})\) which is the centre of the circle (see sidebar). Call centre D.

The radius is \(\sqrt{4985/26}\) (using the distance formula) and the equation is \((x + 3 \frac{3}{26})^2 + (y - 12 \frac{11}{26})^2 = \frac{4985}{26}\)
\[ y = -5x + 28 \text{ (given)} \]

\[ y = -\frac{3}{2}x + \frac{31}{4} \]

\[ 21x^2 + 5x + 1 = 0 \]

\[ (2x - 1)^2 + 3^2 = 1 \]

\[ 11 \times x^2 + 3 \times 3 \]

\[ \text{Substitute } y \text{ into (ii), solve for } x \]

\[ 12 \frac{3}{2} = -5x + 28 \]

\[ 10y = -15x + 77.5 \]

\[ p = 10 \]

\[ 13y = 161.5 \]

\[ \frac{12}{26} = -5x \]

\[ x = -\frac{3}{26} \]

\[ y = 12 \frac{3}{26} \]

\[ \text{Check in (ii) } \]

\[ (x, y) = (0.3, 9) \]

\[ (3, 12.26) \text{ Centre is } \left(-3, \frac{3}{2}, 12.26\right) \]